The Paraconical Pendulum (Allais-Effect) reconsidered

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Abstract

From 2005 to 2007, the Institut für Gravitationsforschung IGF made a series of experiments concerning the so-called Allais-Effect first observed back in 1954 by Nobel-laureate (Economics) Maurice Allais. M. Allais used a pendulum able to rotate freely when extorted along two different axes. The rotation of this kind of pendulum can be calculated and is rather slow. But M. Allais observed this rotation to become stronger especially during the times of solar eclipses. From these observations, he concluded that there must exist an anomaly in space-time or, respectively, gravity and inertia. However, the observations of IGF showed that there is no connection of an Allais-pendulum's rotation with any kind of celestial body's constellations, but only with disturbances in the surrounding of the pendulum. Based on this knowledge, the empirical equation given by M. Allais to describe his phenomenon can be deduced from classical physics.

Der Allais-Effekt

The Allais-Effect [1] as described by M. Allais [2] shows itself most clearly by observing so-called paraconical pendulums.

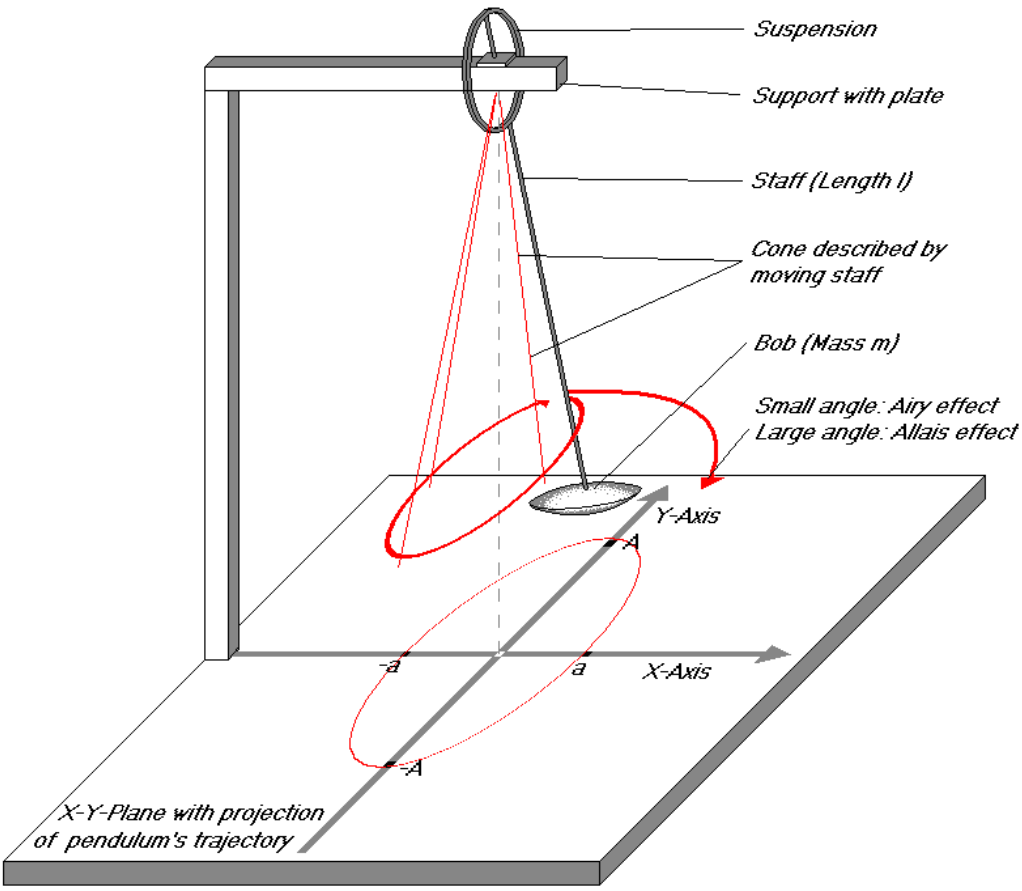

Fig.1) Sketch to show the principal set-up of a paraconical pendulum and to explain the Airy- as well as the Allais-effect. The semi-minor axis of the bob's elliptic trajectory is shown oversized for demonstration purposes.

A paraconical pendulum consists of a heavy bob with mass m –due to its size the bob is often designed as a horizontal lense to reduce air drag during motion- on a short, rigid staff with length l.

The pendulum can freely rotate around the Z-axis –defined as the vertical through the bob's point-of-rest- while oscillating above the X-Y-Plane –with X and Y rectangular to Z, Y pointing to the north and X to the east. The angle φ between an arbitrary orientated line through the origin and the Y-axis is the –nautical- azimuth. Though not a mathematical pendulum, the paraconical pendulum's frequency p and period T are given well enough by the equation used by M. Allais [2, S.55R]:

$$p = \frac{360°}{T} = \sqrt{\frac{g}{l}}$$

The pendulum's free rotation around Z is possible by means of a ring at the upper end of the staff encircling a support. A steel pin at the upper inner wall of the ring rests directly or via a small ball of steel on a plate of a similar hard material.

Due to the weight of the pendulum burdened on a small area of the plate, a dell forms in the plate's surface. Given the shape of the pin or ball, the dell must look like a negative spherical calotte.

If the plate is forced by some disturbances to deviate from the horizontal all time or only from time to time, pin or ball rest in a dell shaped like a negative elliptic calotte.

The dell's shape and the pin's or ball's motion within must directly influence the back-dragging forces F and the oscillation frequency of the pendulum $p = F / ml$ via the short and rigid staff. Only for a dell of spherical calotte shape the back-dragging forces along different directions can be equal or isotropic.

In any other case, the back-dragging forces along different directions must be different or anisotropic.

If a pendulum able to rotate freely around Z is deflected along the Y-axis for a given length A-and the suspenion's pin or ball simultaneously for a small length D in negative Y-direction-, in reality, there will also be a small deflection loosely along the X-Axis for a length $a << A$ -and for the suspension's pin or ball for $d << D$ along the negative X-Axis.

For an isotropic suspension, the total oscillation from these deflections can be divided into two oscillations with the same frequency p.

If projected on the X-Y-Plane, the bob's movement looks like a lengthy ellipse with semi-major axis A and semi-minor axis a. The orientation of the ellipse within the X-Y-plane is defined by the phase angle κ between both extortions. If $\kappa = \pm 90°$, the ellipse is symmetric, in any other cases it is inclined to both the X- and Y-axis.

In motion, the pendulum's staff circumscribes (ancient greek: para) a cone (ancient greek: conos) with a near-elliptic basis what gives the name to this kind of pendulum.

The bob's angular velocity in the X-Y-Plane was first calculated in 1851 by G.B. Airy as [8][9, S.127]:

$$\omega_0 = \pm p \frac{1}{1 + a^2/16l^2} \approx \pm p\left(1 - \frac{a^2}{16l^2} + \frac{a^4}{256l^4} - ...\right) \approx \pm p(1 - a^2/16l^2)$$

This rotation of the bob can be clock- or counter-clockwise.

G.B. Airy also found that this ellipse rotates more or less slowly around the Z-axis, controlled by the difference between the deflections A and a. Defining φ_{Airy} as angle between the semi-major axis A and the Y-axis, the angular velocity of the rotation caused by this Airy effect is given by [8][9, S.127]:

$$\omega_{Airy} = \frac{d\varphi_{Airy}}{dt} = \omega_0 \frac{3}{8}\frac{A}{l}\frac{a}{l} \approx \pm p(1 - a^2/16l^2)\frac{3}{8}\frac{A}{l}\frac{a}{l}$$

This rotation will always have the same direction as the rotation of the bob itself.

But ω_{Airy} is small: a pendulum with $l = 1m$, $A = 0.08m$ and $a \le 0.001m$ will take at least up to 66663 s or 18 h 31 min. for a full rotation of the large A around Z.

For a pendulum free to rotate around Z, the Coriolis effect has also to be taken into account. This effect causes rotations of A in dependence of the observation place's latitude L. The semi-major axis A rotation with angle φ_{Cor} caused by $\omega_{Erde} = 4.167 \cdot 10^{-3}\text{°}/s$ has an angular velocity of:

$$\omega_{Cor} = \frac{d\varphi_{Cor}}{dt} = -\omega_{Erde} \sin[L]$$

This rotation will always be clockwise.
In Europe –latitude $L \approx 50°$ N- a full Coriolis rotation takes 112787 s or 31 h 20 min.
The rotation of the semi-major axis A with an (azimuth-)angle φ relative to Y caused by Airy- and Coriolis-Effect together has the angular velocity:

$$\omega = \frac{d\varphi}{dt} = \omega_{Cor} + \omega_{Airy} = -\omega_{Erde} \sin[L] + \frac{3}{8}\frac{A}{l}\frac{a}{l}\omega_0$$

Now M. Allais seems to think, that the Airy-Effect is largely caused by an anisotropic suspension of a pendulum. Therefore, he tried to build paraconical pendulums with near-isotropic suspensions by using a very hard material (Achat, 7 on Mohs scale of 10) for the plates the pendulum staffs' pins or balls rest on. With this, he aimed at reducing the depth of dells in plates and, therefore, the anisotropy of pendulums' suspensions.
In M. Allais' view, his altered, near-isotropic paraconical pendulums did not show the Airy effect in any measurable way. The rotations of their semi-major axis' A should only be subjected to the Coriolis effect, making their angular velocity equal to:

$$\omega = \frac{d\varphi}{dt} \approx \omega_{Cor} \approx -\omega_{Erde} \sin[L]$$

But several series of observations –mostly weeks of observations, formed by a multitude of 12 min. intervals- with M. Allais' modified pendulums during the years from 1954 to 1960 showed, that sometimes and especially in times of solar eclipses the rotations showed a much higher angular velocity than M. Allais had anticipated. This phenomenon was called Allais effect.
As a result of his experiments, M. Allais gives the following, empirical equation describing the angular velocity of a Paraconical Pendulum's large semiaxis' A rotation by [2, S.31R]:

$$\omega = \omega_{Cor} + \omega_{Allais} = -\omega_{Erde} \sin[L] \pm k \sin[2(\chi - \varphi)]$$

The last term on the right:

$$\omega_{Allais} = \frac{d\varphi_{Allais}}{dt} = \pm k \sin[2(\chi - \varphi)]$$

is referred to as Allais effect term in this text.
Unfortunatly, M.Allais does not define parameter k. Parameter χ in the Sinus, following M. Allais, is the azimuth of his effect on Earth's surface.
M. Allais interprets his effect not as an artefact of anisotropic pendulum suspensions, but as a modified Airy effect caused by anisotropic terrestrial, lunar and solar gravity fields or, respectively, spatial curvatures.
Please notice: M. Allais here omits altogether, that the Airy-Effect -and therefore also his own Allais-Effect- is just caused by a single disturbance at the start of a pendulum and must therefore remain constant during the whole oscillation of this pendulum.
But M. Allais uses these effects to explain observations during time spans of at least 12 min, while rotation angular velocities were changing. This is a contradiction.
However, a change of space-time or gravity caused by certain constellations of celestial bodies demonstrated by a quite simple experiment is noteworthy.
Therefore, IGF started to make measurements with a paraconical pendulum back in 2005 and continued to work on this topic until 2007 [3][4][5].

The IGF Allais-Effect-experiment

The paraconical pendulum used by IGF had a staff length of l = 0.9 m. It's period was measured as $T = 1.8587s$ (1.9s calculated), it's frequency as $p = 193.6838°/s$ (189.5°/s calculated).

The initial deflection in Y-direction $A = 0.08m$ was the same for all oscillations. Because the Coriolis effect was clearly visible for some minutes past the start of an oscillation, and, therefore, must have been much stronger than the Airy effect, one can conclude that the initial deflection a was $a \leq 0.0005m$. The phase angle κ was about $\kappa \approx \pm 90°$, because during the first oscillations of the pendulum A remained nearly parallel to Y [3].

To have a pendulum able to rotate freely around Z, a near-isotropic suspension as described by M. Allais was used. However, in spite of using a very hard plate (Annealed Steel, 8 on Mohs scale of 10), dells of 25 µm depth and some 10ths µm in length and breadth could not be avoided, as electron-microscope-images showed [4].

Due to the suspension via a ring on the pendulum's staff, the rotation angle φ of the semi-major axis A was limited to $\varphi_{Max} = \pm 30°$. If the inner edge of the ring came near it's support, the pendulum was stopped, brought back into start position, deflected and let go. This took about 3 min. An observation interval could, therefore, take 57 min. only. To avoid errors, only measurement cycles of 30 min. duration were done.

The measurement of the semi-major axis' rotation angle φ was done by using two laser beams. One –of a laser rangefinder- measured the pendulum's motion at the lower end of the staff beneath the bob, the other one worked at the upper end off the of the suspension ring via a small mirror projecting the beam on a photodiode scale at a nearby wall. Both values for φ were in good accordance. The largest error was $\Delta\varphi = 0.00333°$ [3].

For about 3.9 min. following the start of the pendulum, the semi-major axis A rotated clockwise. This was the manifestation of the Coriolis effect. At the location of IGF at a latitude of 50° N , this rotation's angular velocity is $\omega_{Cor} = -\omega_{Erde} \sin[L] = -0.0032°/s$.

As stated before, the Airy effect rotation angular velocity must by then have been constant and smaller than $\omega_{Airy} = 0.0032°/s$.

Towards the end of the first 3.9 min. the effect described by M. Allais occurred in ω with ω_{Allais} . After that time, ω_{Allais} was dominating ω. Fig.2) shows a typical $\varphi - t$ -Measurement with the IGF paraconical Pendulum.

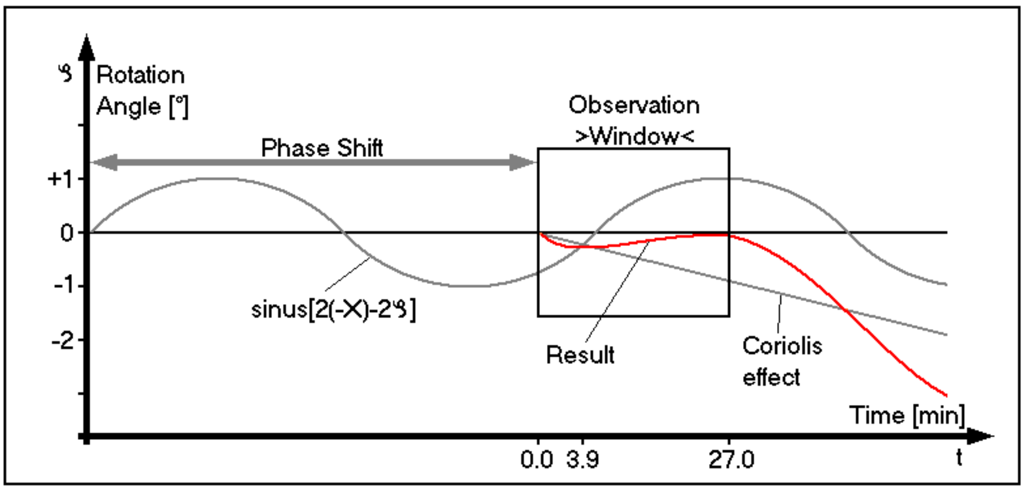

Fig.2) Sketch of a measurement of φ versus t with the IGF paraconical pendulum

This paper is based on the results of measurements with the IGF paraconical pendulum made from Feb. 2005 to Feb. 2007, covering a time span of about 2 years.
During these years, one total solar eclipse (29.3.2007), one circular solar eclipse (3.10.2005) and two partial lunar eclipses (14.3.2006, 7.9.2006) occurred, which were visible at Germany. In any of these case, Sun, earth and Moon nearly stood in a line.
Because the IGF set-up was automatized and small-scale Allais-effects were expected for simple Sun-Earth-, Sun-Moon- or Earth-Moon-constellations, continuous measurements were performed during these two years. Constellations with Jupiter were also noted since during an experiment by M. Allais in 1954, Jupiter lined up together with Sun, Earth and Moon [3].
Concerning solar and lunar eclipses, no significant changes of $\Delta\varphi$ or $\Delta\omega$ per time t were recorded when compared with similar measurements one or two days before or after these events. The earlier results of M. Allais could not be confirmed during all of these 4 celestial events [3][4].
Other constellations which occurred during two years of continous measurment at IGF showed a more detailed picture of significant changes $\Delta\varphi$ or $\Delta\omega$ per time t, but no other result. For example, a measurement cycle of about 50 days showed these coincidences of sudden changes of $\Delta\varphi$ or $\Delta\omega$ per time t with zenith passages of Sun, Jupiter, and Moon:

Celestial Body	Zenit Passage	Correlated with Δφ-t		Uncorrelated with Δφ-t	
Sun	50	20	40.00 %	30	60.00 %
Moon	48	17	35.42 %	31	64.58 %
Jupiter	50	22	44.00 %	28	56.00 %
Jupiter-Moon	1	8	42.11 %	11	57.89 %
Sun-Moon	1	6	42.86 %	8	57.14 %

Only 40.88 % of the observed events with changes $\Delta\varphi$ or $\Delta\omega$ per time t seem to be linked with celestial events. The bigger part of changes $\Delta\varphi$ or $\Delta\omega$ per time t, -59.12 %- shows now connections to celestial events whatsoever. Again this is in contradiction with the earlier results of M. Allais [3].
Without some correlating influence of celestial bodies it is, of course, quite unlikely for two identical paraconical pendulums at different locations to show the same behaviour. Experiments of this kind by M. Allais himself in St. Germain en Laye and Bougival in 1958 did, therefore, show no positive results.
The solar eclipse in 2005 was also accompanied by measurements with two Paraconical Pendulums of the same type located at IGF near Aschaffenburg and Praga. The results of changes $\Delta\varphi$ or $\Delta\omega$ per time t showed no similarities, confirming thereby the negative result of M. Allais of 1958 [4].
However, the rotations of the IGF pendulum can be explained rather well by disturbances of the soil and the IGF building [4]. The vibrations of the building, which are recorded in the institute, are coincident at least in times, if not in strength, with more or less significant changes of $\Delta\varphi$ or $\Delta\omega$ per time t in the pendulum's behaviour. This is, of course, caused by the structure of the building and the soil below it [5].
Even larger earthquakes –which luckily occurred far away- were recorded, so for example the one in Greece end of June 2007 and the one in the Japanese Sea in July 2007 [3]. Another one was detected in October, 23rd, 2006, when, at a distance of about 10 km to the institute, an old bomb of WW II exploded and killed one person [6].
All these facts disapprove the conclusions of M. Allais. It is very probable that M. Allais was also measuring only vibrations in his surroundings. Because of the co-incidence of about 40% between celestial constellations and observed vibrations, he was erronously led to his conclusion of a new type of gravity and spatial anisotropy.

Explanation of Allais-Effect by classical physics

One has to suppose that M. Allais as well as IGF did only observe effects of classical pendulum motion. Therefore, the empirical equation of M. Allais [2, S.31R]:

$$\omega = -\omega_{cor} \sin[L] \pm k \sin[2(\chi - \varphi)]$$

may be derived from classical physics.

Due to a paraconical pendulum's short, rigid staff the oscillations of the pendulum's bob are directly connected to the motion of the suspension's pin or ball in the dell of the plate on the support. As described before, the deflections of the bob A und $a << A$ with Phase κ near $\kappa \approx \pm 90°$ have their counterparts in the extortions of pin or ball in the suspension plate's dell. Just small disturbances generating a deviation of this plate from the horizontal will make the dell look elliptic for pin or ball. In this case, the reset forces and, then, the frequencies of an oscillatoric motion of pin or ball in the dell p_D and p_d and, therefore, for the pendulum p_A and p_a, are different [10].

Assume a series of disturbances in Y-direction, described as one, periodic influence with a certain frequency P. Then [7]:

$$p_d = p$$

$$p_D = p_d + P = p + P$$

The equations of motion for pin or ball in their dell are – for $\kappa \approx \pm 90°$ - with extortion D in Y- and extortion $d << D$ in X-direction:

$$d(t) = d \cos[pt]$$

$$D(t) = D \cos[pt + Pt]$$

The radius r of the elliptic motion of pin or pall in the suspension plate's dell is:

$$r^2 = d^2 + D^2$$

Differentiation of r^2 gives:

$$\frac{dr^2}{dt} = 2d(t)\frac{dd(t)}{dt} + 2D(t)\frac{dD(t)}{dt}$$

Inserting the equations for $D(t)$ and $d(t)$ gives:

$$\frac{dr^2}{dt} = 2p\left(d^2 \sin[2pt] + D^2 \sin[2(pt + Pt)]\right)$$

Because d is much smaller than D, this equation can be simplified:

$$\frac{dr^2}{dt} \approx 2p\left(D^2 \sin[2(pt + Pt)]\right)$$

Since r^2 has quite the same size than D^2:

$$\frac{dD^2}{dt} \approx 2p\left(D^2 \sin[2(pt + Pt)]\right)$$

The semi-major axis' D rotation's angular velocity can be found by dividing this equation by D^2. This rotation may be clock- or counter-clockwise.

$$\omega_{Liss} = \frac{d\varphi_{Liss}}{dt} \approx \pm 2p \sin[2(pt + Pt)]$$

Of course, this is nothing else than an equation describing the Lissajous effect [10]: pin or ball motion in the dell of the suspension's plate is elliptic, and the bob's motion at the other end of the pendulum's rigid staff, therefore, must be, too. Only the deflections are larger.

When influenced by disturbances with a frequency P from the pendulum's surroundings, the suspension plate gets inclined and the dell gets elliptic. The elliptic movement of pin or ball must not follow a closed trajectory like an 0, but will perform motions like an α, ∞ or

something even more complicated which is commonly known as Lissajous figure, depending only on the value of P.
P largely depends on disturbances in the surroundings of a paraconical pendulum, mainly of smaller or bigger vibrations of the soil. Of course, vibrations are kept as small as possible when performing experiments with a paraconical pendulum. But sometimes the motion of two persons at one time in a building may cause enough vibrations to force a paraconical pendulum to show a strong rotation. At other times, however, this will not happen and the rotation remains small.
Taking $Pt \approx 90° + \chi$ with time-dependent χ and using $k = 2p$ as well as $\varphi = pt$, the equation for the Lissajous effect transforms to:

$$\omega_{Liss} = \pm 2p \sin[2(pt + Pt)] = \pm k \sin[2(\chi - \varphi)]$$

It is exactly the same as the equation for the Allais-Effect:

$$\omega_{Allais} = \pm k \sin[2(\chi - \varphi)]$$

Taking the Coriolis-Effect into account, one obtains:

$$\omega = \omega_{Cor} + \omega_{Liss} = \omega_{Cor} + \omega_{Allais} = -\omega_{Erde} \sin[L] \pm k \sin[2(\chi - \varphi)]$$

This equation is applicable to isotropic as well as anisotropic pendulums.
The Airy-Effect as referred to by M. Allais does not occur. Certainly, it existed in all experiments done by M. Allais and IGF, but was much too small to be visible aside the Coriolis and Lissajous effects.
Contrary to the Lissajous effect, the Airy effect was never capable to explain the observations of M. Allais: it remains constant after starting a pendulum. Due to P, the Lissajous effect can react on events while a pendulum performs it's oscillation, which is just what observations of paraconical pendulums showed.
The Airy effect even has no connection to the isotropy or anisotropy of a pendulum as supposed by M. Allais:

$$\omega_{Airy} \approx \pm p(1 - a^2/16l^2)\frac{3}{8}\frac{A}{l}\frac{a}{l}$$

Te equation reveals only a connection to the size of the deflections A and a, but not to other constant or time-dependent disturbances in a pendulum's suspension.
The Allais effect is just the performance of Lissajous figures by a paraconical pendulum because of disturbances in its suspension. The short, rigid staff directly transmits the behaviour of the suspension to the pendulum's bob (This is also why a Foucault Pendulum behaves otherwise: it has no rigid staff.).
The disturbances leading to the Allais or, more appropriate, the Lissajous effect are erratic and weak most of the time: a bypassing pedestrian, a distant earthquake etc.[5]. However, in times of solar eclipses with many persons under way to observe them, disturbances may get strong. And so, a paraconical pendulum reacts strongly. All this explains the IGF observations on the paraconical pendulum.

Conclusion

The so-called Allais effect known from observations of paraconical pendulums has nothing to do with an up to now unknown change of gravity fields during certain celestial constellations.
The experimental as well as the theoretical examination of IGF shows clearly, that the Allais effect can be explained by the well-known Lissajous effect. This effect arises because of all-day erratic disturbances and vibrations in the surroundings of a paraconical pendulum.

Acknowledgement

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